30.67. Model: Capacitance is a geometric property.
Visualize: Please refer to Figure P30.67. Shells R_1 and R_2 are a spherical capacitor C. Shells R_2 and R_3 are a spherical capacitor C'. These two capacitors are in series.
Solve: The ratio of the charge to the potential difference is called the capacitance: $C = Q/\Delta V_C$. The potential differences across the capacitors C and C' are

$$\Delta V_C = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_1} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_2} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\Delta V_{C'} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_2} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_3} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R_2} - \frac{1}{R_3}\right]$$

$$\Rightarrow C = (4\pi\varepsilon_0)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)^{-1} \qquad C' = (4\pi\varepsilon_0)\left(\frac{1}{R_2} - \frac{1}{R_3}\right)^{-1}$$

Because these two capacitors are in series,

$$\frac{1}{C_{\text{net}}} = \frac{1}{C} + \frac{1}{C'} = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_2} - \frac{1}{R_3}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{R_3 - R_1}{R_1 R_3}\right)$$

$$C_{\text{net}} = 4\pi\varepsilon_0\left(\frac{R_1 R_3}{R_3 - R_1}\right) = \frac{1}{9.0\times10^9\ \text{N m}^2/\text{C}^2}\left[\frac{(0.01\ \text{m})(0.03\ \text{m})}{0.03\ \text{m} - 0.01\ \text{m}}\right] = 1.67\times10^{-12}\ \text{F} = 1.67\ \text{pF}$$

Assess: C_{net} depends on only the inner and outer shells, not on R_2.